

Fermat to Frenicle*

Translated by Jason Ross

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The nature of every non-square number is that there exist an infinite number of squares, which when multiplied by the non-square and unity is added, give a square number.

Example: 3 is a non-square, which, when multiplied by 1, which is a square, makes 3, and adding unity, makes 4, which is square.

The same 3, multiplied by 16, which is a square, makes 48, and adding unity, makes 49, which is square.

Similarly, there are an infinite number of squares which, multiplied by 3 and adding unity, make a square number.

I ask you to find a general rule where, given a non-square number, you can find the square numbers which multiplied by it with one added make squares.

For example, what is the smallest square, which multiplied by 61 and adding 1, makes a square?

What is the smallest square, which multiplied by 109 and adding 1, makes a square?

If you do not send the general solution, send me the particular answers of the two numbers that I have chosen (if you do not want too much work).

After having received your response, I will pose another question to you. It seems, without saying it, that my proposition is only for finding whole numbers which satisfy the equation, for, in the case of fractions, the least of arithmeticians would quickly come to an answer.

*This is an extract first sent by Cl. Mylon to Huygens at the end of a letter dated Mar 2, 1657; Huygens then sent it to Schooten on Mar 9.