

Philosophical Fragments

Bernhard Riemann

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With each simple act of thought, something enduring, substantial, enters into our soul. This substantial thing appears to us, indeed, as a unity; however, it appears (insofar as it is the expression of a spatial and temporal extension) to contain an inner manifoldness; hence, I call this a “*thought-object*” [“*Geistesmasse*”]. — All thought is, according to this, the formation of new thought-objects.

The thought-objects entering into the soul, appear to us as conceptual representations [*Vorstellungen*]; the distinct inner state of each [conceptual representation] determines its distinct quality.

The currently forming [*sich bildenden*] thought-objects merge, blend, or entangle themselves to a certain degree, partially with one another, partially with older thought-objects. Both the type and strength of this union depend on conditions recognized only in part by HERBART, which I will elaborate in what follows. They rest, principally, on the inner relationship of the thought-objects.

The soul is a compact thought-object, bound together in the most intimate and most manifold way. It constantly grows by the introduction of thought-objects, and upon this rests its further development.

Once formed, the thought-objects are imperishable, their blending indissoluble; only the relative strength of their unions is changed by the addition of new thought-objects.

Thought-objects require no material carrier for their continued existence, and exert no lasting effect upon the world of phenomena [*Erscheinungswelt*]. Thus, they stand in no relation to any part of matter and, consequently, have no position in space.

On the other hand, all beginning, generation, all formation of new thought-objects, and all unification of the same, requires a material carrier. Hence, all thinking comes to pass at a determined place.

It is not the retention of our experience, but only thinking, that requires effort, and the exertion of force is, as far as we can gauge, proportional to the mental activity.

Each entering thought-object stimulates all related thought-objects, and indeed all the more strongly, the less their inner compositions (quality) differ.

This stimulation is limited, however, not merely to the related thought-objects, but is also extended mediately [*mittelbar*] to those with which they are connected [*zusammenhängenden*] (i.e. bound [*verbunden*] with them in earlier

thought processes). Thus, if among the related thought-objects, a portion of them are connected to one another, then the latter will become stimulated, not only immediately, but also mediately, and, hence, [stimulated] relatively more strongly than the rest.

The interaction of two simultaneously formed thought-objects is determined by a material process [*materiellen Vorgang*] connecting the places where they are formed. Likewise, it is due to material causes, that all currently forming thought-objects enter into immediate interaction with [those thought-objects] formed immediately before; it is mediately, however, that all older thought-objects connected to these are stimulated to activity, and indeed all the weaker, the more distant their connection with them, and the less they are connected among themselves.

The simplest and most general expression of the activity of older thought-objects is reproduction, which consists in the acting thought-object striving to generate one similar to itself.

The formation of new thought-objects rests on the common activity partly of the older thought-objects, partly of material causes, and indeed is hindered or promoted by each commonly acting [thought-object] according to the inner dissimilarity or similarity of those thought-objects which it strives to generate.

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The form of a thought-object being formed (or the quality of the conceptual representation accompanying its formation) depends on the relative form of motion of the material, in which it is formed, so that an identical form of motion of the material implies an identical form of thought-object formed in it and, inversely, an identical form of thought-object presupposes an identical form of motion of material, in which it is formed.

The totality of the simultaneously forming (in our cerebro-spinal system) thought-objects are interconnected in consequence of a physical (chemical-electrical) process between the places where they are formed.

Each thought-object strives to generate a similarly formed thought-object. It thus strives to produce that form of motion of the material with which it was formed.

The assumption of a soul as a unified carrier of that which persists [*des Bleibenden*], which is generated in individual acts of soul-life¹ [*Seelenlebens*] (conceptual representations), is supported

1. by the close connection and the interpenetration of all conceptual representations. However, in order to clarify the connection of a specific new representation with another one, the assumption of a unified carrier alone is *not sufficient*; rather, the reason for why they go into just this specific connection in this specific strength, must be sought in the representation with which they are combined. *Besides* these reasons, however, the assumption of a unified carrier of all representations is superfluous....

¹This is a term from Fechner - *trans.*

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We now apply these laws of mental processes, which we have been led to by the explanation of our own inner perception, to the explanation of the purposefulness perceived on the Earth, i.e. to the explanation of existence and of historical development.

In order to explain our soul-life, we must assume that the thought-objects produced in our nervous system endure as a part of our soul, that their interconnections continue unchanged, and that they are subjected to a change only insofar as they enter into a connection with other thought-objects.

An immediate consequence of these defining principles is that the souls of organic beings, i.e. the compact thought-objects originating during their lives, also persist after death. (Their isolated continuance [persistence as isolated entities] is not adequate.) However, in order to explain the development of organic nature according to plan, in which the experiences collected earlier apparently served later creations as the foundation, we must assume that these thought-objects enter into a larger, compact thought-object, the Earth Soul, and there serve a higher soul-life according to the same laws as the thought-objects produced in our nervous systems serve our own soul-lives.

Thus, as e.g. upon viewing a red surface, the thought-objects produced in a collection of individual primitive fibrils [*Primitivfasern*] ² are connected to a single compact thought-object, and thus enter into our thoughts simultaneously, so also, the thought-objects produced in the various individuals of a plant species, which enter into the Earth Soul from a climactic region of the Earth's surface, combine themselves into a total impression. Just as the various sensory impressions of the same object are united into an image of that object in our soul, so will all of the plants from a part of the Earth's surface give the Earth Soul an image, elaborated to the finest degree, of the climatic and chemical state of that region. In this way, it is explained how the plan of later creation is developed out of the earlier life of the Earth.

However, according to our defining principles, the persistence of existing thought-objects is in no need of a material carrier, but all connection between them, at least all connection of different thought-objects, can occur only by means of new thought-objects produced in a common nervous system.

For reasons which should be developed later, we can search for the substrate of a mental activity only in ponderable matter.

Now, it is a fact that neither the rigid terrestrial crust, nor all ponderable [matter] on it, serve a common mental process, rather, the movement of these ponderable masses must be explained by other causes.

Accordingly, all that remains is the assumption, that the ponderable masses within the solidified terrestrial crust are carriers of the Earth's soul-life.

Are these suited to do so? What are the external requirements for the possibility of the life process? The universal experiences about living processes available to our observation must form the foundation for it; but only insofar

²See Robert Remak, who wrote a book on *Primitivfasern*, and taught at the University of Berlin when Riemann went there. He also defined the three embryonic germ layers.

as we succeed in explaining them, can we draw conclusions which are thus applicable to other circles of phenomena.

The universal experiences about the external requirements of the life process in the sphere of phenomena available to us are:

1. The more highly and more completely developed the life process, the more the carrier itself requires protection against outer motive causes, which strive to change the relative position of the parts.

2. The physical processes known to us (metabolism), which serve as a medium for the thought process:

a) absorption of elastic fluids through liquid fluids.³

b) endosmosis.

c) formation and destruction of chemical bonds.

d) galvanic currents.

3. The material in organisms has no recognizable crystalline structure, it is partially firm (not very brittle), partially gelatinous, partially liquid or elastic fluid, but always porous, i.e. notably permeable to elastic fluids.

4. Among all chemical elements, there are only the four so-called organic general carriers of life processes, and of these there are again entirely determined combinations, the so-called organizing elements of organic bodies (protein, cellulose, etc.).

5. The organic combinations exist only up to a definite upper temperature limit, and only to a definite lower limit can they be carriers of the life process.

ad.⁴ 1. Changes in the relative position of the parts will, in gradually smaller degrees, be produced by mechanical power, by temperature changes, by light rays; accordingly, we can order the facts, whose general expression is our theorem, as follows:

1. The capability of lower organisms to propagate through division. The constantly decreasing ability of higher animal organisms to reproduce.

2. The parts of plants are more sensitive to temperature changes, the more intensive and the more highly developed the life process is in them. In higher animal organisms, an almost constant temperature reigns, and indeed most completely in their most important parts.

3. The parts of the nervous system, which serve independent thought activity, are the most protected possible against all of these influences.

The fact first enumerated clearly has its foundation in the fact that the relative position of the parts can be determined the sooner by the processes inside the material, the less they are determined by external motive causes. This independence from external motive causes, however, happens within the terrestrial crust to a far higher degree, than could ever be attained by organic apparatuses [*Einrichtungen*] outside the terrestrial crust.

What is collected together under 4 and 5, when considered in relation to the following facts, is seemingly against our assumption; in fact, it would be so, if

³Here, by *elastic* and *liquid* fluids, Riemann seems to mean what we would today call *compressible* and *incompressible* fluids.

⁴*a dato*, from the given — *trans*.

an absolute validity were attributed to what we perceive as being the conditions for the possibility of a life process, and not merely a validity relative to our sphere of knowledge. The following reasons, however, speak against the former:

1. With the exception of the Earth's surface, the whole of nature must be considered as dead, since heat and pressure relations are predominant on all other celestial bodies, under which organic compounds cannot exist.

2. It is absurd to assume that the organic had arisen from the inorganic on the rigid terrestrial crust. In order to explain the emergence of the lowest organisms on the terrestrial crust, an organizing principle, therefore a thought process, must be preassumed under conditions where organic compounds could not exist.

Hence, we must assume that these conditions are valid only for the life process under the current relationships on the surface of the Earth, and only in so far as we succeed in explaining them, can we judge therefrom the possibility of the life process under other relationships.

Why are only these four organic elements the universal carriers of the life process? The reason can be sought for only in properties through which these four elements are differentiated from all others.

1. One such common property of these four elements is found in the fact that they, and their compounds with all other materials, are most difficult to condense, and a portion of them are, to this day, impossible [to condense].⁵

2. Another property common to them, is the great multitude of their compounds and their easy decomposability. However, this property could just as well be a consequence of, as be a reason for, their use in living processes.

However, that the first property, of being difficult to condense, makes these four elements preeminently suited to serve living processes, will to some extent already be explainable immediately from the actual requirements of the life process, collected under 2 and 3, yet more, however, if the phenomena pertaining to the condensation of gas into liquid fluids and solid bodies are sought to be reduced to causes...

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Zend-Avesta, in fact, a life-making word, creating new life for our spirit in thought and in belief; for, like many thoughts, which, indeed once operated powerfully in the course of human development, it remained in us only through tradition, it now arises suddenly, from its apparent death, to new life in a more pure form, unveiling new life in nature. For, how immeasurably the life of nature is opened before our eyes, that which hitherto has been made known only on the surface of the Earth, now appears so inexpressibly more sublime than before. What we considered to be the seat of senselessly and thoughtlessly acting forces, now appears as the workplace of the highest spiritual activity. In a wonderful way, the goal is fulfilled which our great poet portrayed in prescient rapture, and which swam before the mind of the prescient investigator.

⁵on condensation of hydrogen, carbon, oxygen and nitrogen

Just as FECHNER sought to demonstrate in his *Nanna* that plants are endowed with soul, so also the starting point of his considerations in *Zend-Avesta* is the teaching that stars are endowed with soul. The method that he made use of, is not the abstraction of general laws through induction, and their application and proof in the explanation of nature, but rather analogy. He compares the Earth with our own organism, which we know possesses a soul. Therewith, he seeks not merely one-sidedly for similarities, but, rather, also does equal justice to the dissimilarities, and comes thus to the result, that all similarities indicate that the Earth were a being endowed with soul; from all the dissimilarities, however, he concludes that it were a much higher order ensouled being than we. The persuasive power of this representation lies in its universal, detailed elaboration. The total impression of the image of the life of the Earth unfurled before us, must provide evidence for the hypothesis, and makes up for what the individual conclusions lack in rigor. This evidence rests essentially on the clearness of the image, on its greatest possible execution into the details. Hence, I believe I would damage the FECHNERIAN hypothesis, if I sought to lay open in excerpt the path which he took in his work. In the following discussion of the FECHNERIAN hypothesis, I will disregard the form in which it was carried out, and only consider its substance, and will rely upon the first method, the abstraction of general laws through induction and their proof in the explanation of nature.

We first ask: from what do we conclude that a thing possesses a soul (the existence of an enduring unified thought process in it)? We are immediately certain of the existence of our own soul, while, with others (men and beasts), we conclude [the same] from their individual purposeful movement.

Whenever we reduce the well-ordered purposefulness [*Zweckmässigkeit*] to a cause, we seek this cause in a thought process; we have no other explanation. However, in any event, I can only consider thought to be a process in the interior of ponderable matter. The impossibility of using the spatial motion of matter to explain thoughts, will indeed be clear to everyone through an unbiased decomposition of inner perception; yet, the abstract possibility of such an explanation may be conceded here.

That purposefulness is perceived on the Earth, nobody will deny. It is therefore asked: Where must we place these thought processes, which cause this purposefulness?

The discussion here is only about contingent goals (occurring in bounded space and time); non-contingent [necessary] goals find their explanation in an eternal⁶ will (not produced in a thought process). The only purposefulness, whose cause we perceive, is the purposefulness of our own actions. It springs from the will towards the goal and from consideration of the means.

If we now find a body consisting of ponderable matter, in which a system of continuing purpose and effect relations comes completely to conclusion, then, to explain this purposefulness, we can assume a continuing unified thought process;

⁶The word “eternal” [*ewig*] appears in the philosophical fragments published among Riemann’s collected works, by Heinrich Weber. In Riemann’s handwritten notes appears, instead, the word “anonymous” [*anonym*]. — *trans.*

and this hypothesis will be the most probable, if 1) these purposefulnesses do not come to conclusion in the parts of the body, and 2) no reason exists to seek its cause in a greater whole, which belongs to the body.

If we apply this to purposefulness observed in men, beasts and plants, then it arises that a portion of these purposefulnesses are explained by the thought processes within these bodies, but another portion, the purposefulness of the organism, is explained from a thought process in a greater whole.

The reasons for this are:

1. The purposefulness of biological structures [*organische Einrichtungen*] does not find its fulfillment in a single organism. The reasons for the structure of the human organism are clearly to be sought in the constitution of the whole surface of the Earth, organic nature included.

2. Organic motions repeat themselves innumerable, both in different individuals simultaneously, and in the life of an individual or of a generation, one after another. In each case, therefore, there is not any particular, but one common cause to be assumed, for the purposefulness which already lies in them of its own accord.

3. Biological structures, both in the lives of particular individuals (with humans and animals), and in the lives of particular species (with plants and embryos) undergo no development. The cause for their *purposefulness* is not to be sought for in a simultaneously progressing thought process [of the individual or of the species].

After removal of this (organic) purposefulness, there still remains, as is generally recognized with humans and animals, and with plants in FECHNER's opinion, a closed system of interlocked, variable purpose and effect relations; and this purposefulness in them is explained by a unified thought process.

These conclusions from our principles are confirmed by our inner perception.

By the same principles, however, we must search for the cause of the purposefulness, perceived in organisms, in a unified thought process in the Earth, for the following reasons:

a) The purpose and effect relations in organic life on the Earth are not decomposable into individual systems, but rather they are all interlocked. They can therefore not be explained by several distinct thought processes in parts of the Earth.

b) There is, as far as our experience extends, no reason to seek the causes of these purposefulnesses in a greater whole. All organisms are determined solely for life on the Earth. The condition of the Earth's crust therefore contains the entire (external) reason for their structure.

c) They are unique. According to everything that experience teaches us on this, we must assume that they do not recur on other heavenly bodies.

d) They do not remain for the [entire] life of the Earth. Rather, in its course, there appear ever newer and more perfect organisms. We must thus seek the cause in a simultaneous thought process progressing to higher degrees.

From the standpoint of exact science — the explanation of nature from causes — the assumption of an Earth Soul is thus an hypothesis for explanation of the existence and historical development of the organic world.

1) But that, how?

Thought is a process within ponderable matter. Our external experience, the facts of our external perception, which must find their explanation in the processes within ponderable or gravitating matter, are

1) universal gravitation 2) the universal laws of motion.

Something persisting underlies each act of thought, something which, however, is manifested only under the specific occasion of memory as such, without exerting any enduring influence upon phenomena. Therefore with each act of thought, something persisting enters our soul, something which exerts no enduring influence upon phenomena.

On the other hand, our external experiences about ponderable matter can be explained if it is assumed that a homogenous substance fills the whole of infinite space, and constantly flows into ponderable matter and vanishes.

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The effects of ponderable matter upon ponderable matter are

1) attractive and repulsive forces inversely proportional to the square of the distance

2) light and radiant heat.

Both classes of phenomena can be explained if it is assumed that the entirety of infinite space is filled evenly by a homogenous substance, and that every particle of substance only influences its immediate environment.

The mathematical law according to which this happens can be thought of as decomposed

1) into the resistance, with which a particle resists a change in volume, and

2) into the resistance, with which a physical line element resists a change in length.

Upon the first part rests gravitation and electrostatic attraction and repulsion; upon the second, the propagation of light and heat, and electrodynamic, i.e. magnetic, attraction and repulsion.

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The distinction which Newton makes between laws of motion or axioms, and hypotheses, seems to me untenable. The law of inertia is the hypothesis: If a material point were to exist alone in the world, and to be moved in space with a definite velocity, then it would maintain this velocity perpetually.

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“When the body of the lower soul dies,” says Fechner, “the higher soul takes it up from its life of perception to its life of memory.” The souls of dead creatures ought therefore to form the elements for the soul-life of the Earth.

The different thought processes seem to distinguish themselves primarily through their temporal rhythms. If plants possess souls, then hours and days must be to them what seconds are to us; the corresponding space of time for

the Earth Soul, at least for its outward activity, encompasses perhaps many millennia. As far back as the historical memory of humanity extends, all motions of the inorganic crust of the Earth indeed have yet to be explained by mechanical laws.

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Antinomies

| Thesis | Antithesis |
|---|--|
| Finite, representable. | Infinite, conceptual system lying at the boundary of representability. |
| I | |
| Finite elements of time and space. | The continuous. |
| II | |
| Freedom, i.e. not the ability to begin unconditionally, but rather to choose between two or more given possibilities. | Determinism. |
| In order that decision through free will be possible despite completely determined laws of operation of conceptual representations, it must be assumed that the psychic mechanism itself has, or at least assumes in its development, the property of giving rise to the necessity for free will. | Concerning activity, no one can purport to believe that his own activity plays a role in determining the future. |

III

A temporally acting God (governance of the World).

A timeless, personal, omniscient, omnipotent, all benevolent God (providence).

IV

Immortality.

A Thing-in-itself lying at the basis of our temporal manifestation, equipped with transcendental freedom, inherent evil, and intelligible character.

Freedom is highly compatible with rigorous lawfulness of the course of nature. But the concept of a timeless God is not tenable along with it. Rather, the limitation which omnipotence and omniscience suffers from the freedom of creatures, in the above given meaning, must be lifted by the assumption of a temporally acting God, a ruler of the hearts and fates of men; the concept of providence must be supplemented, and in part replaced, by the concept of governance of the World.

General Relations of Conceptual Systems of Thesis and Antithesis.

The method which Newton applied for the founding of the infinitesimal calculus, and which, since the beginning of this century has been the only one recognized by the best mathematicians, which yields sure results, is the method of limits. Instead of considering a continuous transition from one value of a magnitude to another, or in general, from one mode of determination of a notion to another, this method consists in considering, first, a transition through a finite number of intermediate steps, and then letting the number of these intermediate steps grow, such that the distances between any two successive steps decreases infinitely.

The conceptual systems of antithesis are indeed concepts fixed through negative predicates, but are not positively representable.

For just that reason, because an exact and complete representation of these conceptual systems is impossible, they are inaccessible to direct investigation and treatment by our reflection. They can however be considered as lying at the limits of representability, i.e. a conceptual system can be formed which lies

within the limits of representability, and which passes over into the given system merely via changes in relations of magnitude. Seen from the point of view of relations of magnitude, the conceptual system remains unchanged in the transition to the limit. In the limiting case itself, however, the correlative concepts of the system lose their representability, specifically, those which mediate the connection between other concepts.