Second Challenge to Mathematicians

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Just as soon as someone poses purely arithmetical questions, someone else figures out how to solve them. Is it because Arithmetic has been treated up till now more by means of Geometry than by herself? That is the tendency that appears in the majority of Works, both ancient and modern, and in Diophantus himself. For if it is separated a little more from Geometry than are others, by requiring its analysis to only consider rational numbers, it is still not completely separated, as is overabundantly proved by the *Zetetics* of Viète, in which the method of Diophantus is extended to continuous quantities, and following that, to Geometry.

However, Arithmetic does have a domain proper to itself, the theory of whole numbers. This theory was only slightly outlined by Euclid, and was not cultivated enough by his successors (unless it was contained in those books of Diophantus that the injury of time has deprived us of). Arithmeticians must therefore develop it or renew it.

To clarify their progress, I propose that they demonstrate as a theorem or solve as a problem the following; if they succeed, they will at least know that questions of this type do not concede by their subtlety, difficulty or by their mode of demonstration, to the most celebrated problems of Geometry:[1]

Given an arbitrary non-square number, there are an infinite number of determined squares such that by adding unity to the product of one of them by the given number, a square is formed.

For example, we give 3, a non-square number.

$$3 \times \ 1^2 + 1 = 4 \text{ (square)},$$
$$3 \times 16 \ + 1 = 49 \text{ (square)}.$$

[1] Fermat had trouble getting others interested in Arithmetic!

In place of the squares 1 and 16, one can find an infinite number of other squares satisfying the proposed condition, but I demand a general rule, applicable to any non-square number that can be given.

For example, find a square, such that by adding unity to its product with 149 or 109 or 433, etc., a square is formed.