Fermat to Pascal

August 29, 1654

SIR,

1. Our heated exchanges[1] still continue, and much as you do, I admire the way our thoughts come together so exactly that it seems they were cut from the same cloth and take the same path. Your last Treatises on the *Arithmetical Triangle* and of *Its Application* are an authentic proof: and if my calculations do not err, your eleventh consequence[2] was in the mail from

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[1] *coups fourrés*

[2]The eleventh consequence of the *Treatise on the Arithmetical Triangle* is stated thus: *Each cell of the dividend is double that which precedes in its parallel or perpendicular row.* Pascal called *cells of the dividend* those which the bisector of the right angle of the triangle traverses diagonally: for example G, ψ, C, P, ρ.

G 1	1	1	1	1	1	1	1	1	1
1	ψ 2	3	4	5	6	7	8	9	
1	3	C 6	10	15	21	28	36		
1	4	10	P 20	35	56	84			
1	5	15	53	ρ 70	126				
1	6	21	56	126					
1	7	28	84						
1	8	36							
1	9								
1									

Fermat's proposition on numbered figures is that of the *Observation on Diophantus,* XLVI and in his September 1636 letter to Mersenne, **12**. The eleventh consequence of Pascal's work does not in fact correspond to the first part of Fermat's proposition, namely that

Paris to Toulouse at the same time that my proposition on figurate numbers, which is in fact the same thing, was going from Toulouse to Paris.

I cannot err, if I speak in this way, and I persuaded that the true way to avoid error is to agree with you. But if I were to say more, it would become a compliment, and we have banished this enemy (of mild and easy [trifling] conversations).

It is now my turn to share with you some of my numerical discoveries; but the close of Parliament increases my occupations, and I dare to hope that your kindness will allow me a fair and almost necessary respite.

2. Nevertheless I will respond to your question about the three players who play to two. When the first has one and the others do not, your first solution is the true one, and the division of the money must be as 17, 5, 5: for which the reason is manifest and is always according to the same principle, combinations showing that the first has 17 equal chances while each of the others has only 5.

3. In the future, there is nothing that I will not tell you with complete frankness. Nonetheless, consider, if you think it is appropriate, this proposition:

The square powers of 2, with unity added, are always prime numbers.[3]

The square of 2, with unity added, makes 5, which is a prime number.

The square of the square makes 16, which with unity added is 17, another prime number.

The square of 15 makes 256 which, augmented by unity, makes 257, a prime number.

The square of 256 makes 65,536 which, increased by one, makes 65,537, a prime number.

And so on infinitely.

You may take my work on the truth of this property. Its demonstration is very difficult and I confess that I have yet to find it simply: I would not ask you to solve it, if I had already figured it out.

This proposition serves for the creation of numbers which are to their aliquot parts in given ratios, on which subject I have made considerable

$m(m+1)$ is double the triangle of side m; to find the rest of the proposition in Pascal, further consequences must be added to the eleventh.

[3]Fermat primes. The next number in the series is actually composite, having 641 as a factor. See the "Arithmetic" section at `http://wlym.com/~animations/fermat`.

discoveries. We will speak about it at another time.

I am, Sir, your, etc.,

FERMAT.

Toulouse, the 29^{th} of August, 1654.