

Fermat to Pascal

1654

SIR,

Let us say that I undertake to win a point with a single die in eight rolls; if we agree, after the money is already in the game, that I will not make my first roll, then according to my principle it is necessary that I take out $\frac{1}{6}$ of the pot to be bought out, since I will be unable to roll to win in the first round.

If later we agree that I will not play the second round, then for my indemnity I must be given the sixth part of what remains, namely $\frac{5}{36}$ of the total.

And if after this we agree that I will not play in the third round either, then I must be indemnified by taking the sixth part of what remains, which is $\frac{25}{216}$ of the total.

And if after this we again agree that I will not play in the fourth round, I must take a sixth of what remains, which is $\frac{125}{1296}$ of the total, and I agree with you that this is the value of the fourth round, assuming the preceding payouts were made along the way.

But you proposed in the last example of your letter (I insert your own terminology), that if I endeavor to roll a six in eight rolls, and if I have already made three rolls without succeeding, then if my player proposes that I not at all make the fourth roll, and prefers rather that I retire from the game and leave with him, then $\frac{125}{1296}$ of the pot should go to me.

This is not true, according to my principle. For in this case, the first three rolls having won nothing for the roller, the player who wishes not to make his fourth roll, must take as his indemnity $\frac{1}{6}$ of the total sum remaining in the game.

And if he had played four rounds without rolling the desired face, and then wished not to play the fifth, he would have the same $\frac{1}{6}$ as his indemnity.

For since the entire sum remains in the game, it is not only based on principle, but is in the same sense natural that each roll must confer an equal advantage.

I pray that you let me know whether we are agreed on the principle as I believe we are, or if we differ only in application.

I am, with all my heart, etc.

FERMAT.